Topic: Solving Quadratic Equations by Graphing.

Standards:
21.0 – Students graph quadratic functions and know that their roots are the x-intercepts.
23.0 – Students apply quadratic functions to physical problems, such as the motion of an object under the force of gravity.

As shown on page 2, there are four types of knowledge our students need to be skilled on for a particular mathematical topic:

1. Factual/Memorization
2. Skills/Procedures
3. Concepts
4. Relational

For the topic of solving quadratic equations by graphing, I created a series of activities (see page 3) that cover several of these knowledge types. Specifically, I used the following:

1. ***Memory Game*** (pages 4 - 6) for Factual Teaching/Learning Event .
2. ***Worksheet*** with student work (pages 7 - 8) for Procedural Teaching/Learning Event.
3. ***Task description*** with student work: Part I and Part II (pages 9 - 12) for Conceptual Teaching/Learning Event. This task is Common Core based.

Explanation of Factual – See page 4.

Explanation of Procedural – For the procedural skills, I used a worksheet that also includes a couple of word problems. Students can show their work, and also draw the graph, directly on the worksheet. A worksheet with student work is shown on pages 7 and 8.

Explanation of Conceptual – The conceptual task was an Egg Launch Contest (adapted from www.NCTM.org) the wording of which was modified so that the contest was held at the school where I am student teaching, and the three competing groups were the students of the three Algebra I math teachers at the school. The trajectory of the egg for each of the three competing groups was represented differently (as a table, equation, and graph). Students had to compare data and move between representations discovering the usefulness (or not) of each in determining the roots (zeros).

The task was divided into two parts as a project. In Part I the students worked in pairs to understand the task (page 9) and answer nine questions (pages 10 and 11). This gave the students the opportunity to discuss and collaborate. Part II was a quiz which students answered individually, so I could assess the learning and understanding of each student. Details of the task with student work is shown as Part I on pages 9 (task description), 10 (student work), and 11 (graphs drawn by student), and Part II on page 12 (answers to quiz).

Understanding Knowledge Types

Memor-ization	Skills/ Procedures	Concepts	Relational Knowledge
descriptions, vocabulary, formula recollection	procedures, usually doing something, (verbs)	ideas, understanding things, (nouns)	applications of multiple types of knowledge, synthesis, analysis or evaluation
How This Knowledge is Learned & Retained			
Repeated exposure Memorization Techniques (songs, acronyms, etc.) Drill	Modeling Repeated practice of the same steps with feedback and reinforcement Repeated exposure (same context)	Exploration Inquiry/Discovery Experimentation Hands-on/ Manipulatives Multiple reps connected through writing/discussion Experienced in new contexts Prior to procedures	Exposure to open-ended questions Class/ group discussions Collaboration Authentic experiences
Characteristics of Assessment Questions			
Routine No Context Focus on Recall One Short Answer Closed Quick/seconds	Routine Little or No Context Focus on Procedure and/or Answer One Short Answer Closed Length varies on complexity of skill	Non-routine Can Be In Context Focus on Explanation & Representation Extended Answer Open Middled or Open Ended Medium/ minutes	Non-routine Often In Context Focus on applying knowledge Extended Answer Open Middled or Open Ended Longer/ minutes +

Target Knowledge

Factual	Procedural	Conceptual
1. Quadratic is of the form $ax^2 + bx + c$, where $a \neq 0$. 2. Graph is called a parabola. If $a > 0$, parabola opens upwards. If $a < 0$, parabola opens downwards. 3. Axis of symmetry ($x = -b/2a$) divides parabola into 2 symmetrical halves.	1. Write the quadratic equation in standard form. 2.Calculate the vertex, y-intercept, and zeros 3. Graph the quadratic. 4. Find the x-intercepts (zeros, roots).	Roots, zeros, x-intercept, vertex.

Teaching/Learning Activities

Factual	Procedural	Conceptual
1. Flashcards with definitions. 2. Memory game with multiple opportunities for winning, i.e. getting a matching pair. 3. Poster activity. 4. Interview grid.	1. Teacher puts up problems on board and has (i) a class discussion, and/or (ii) students discussing in pairs and/or (iii) students show answers on whiteboard. 2. Practice problems from textbook for homework, or worksheet in class. For in class worksheet, students can work individually or in pairs and discuss.	1. Quadratic Quandary (adapted from www.lausd.k12.ca.us). Students use quadratic functions that model the height of rockets above the ground after they have been launched to graph the relationship between time and height. Students use the graph to determine the amount of time the rocket stays in the air and then describe how to find and interpret the x-intercepts of any quadratic function. 2. Egg Launch Contest (adapted from www.NCTM.org). Students will represent quadratic functions as a table, graph, and an equation. They will compare data and move between representations discovering the usefulness (or not) of each in determining the roots (zeros).

Memory Game for Factual Information

I created a ***memory game with two twists*** to cover the factual information described in the topic plan summary. The two twists are described below:

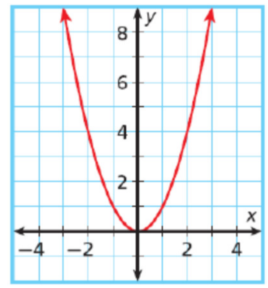
> [1] **Multiple Opportunities for Success:** Instead of only two cards being a match for each other, I created three groups of four cards (making a total of 12 cards), with each group embodying a mathematical concept. In each of the three groups of four, each card could be a match for any of the other three cards. This way a player has *multiple opportunities for succeeding* (getting a matching pair). The three groups of four cards are:
>
> - roots; zeros; solutions; x-intercepts
> - parabola; you-vee shape; curve of a 2^{nd}-degree equation; graph of $ax^2 + bx + c = 0, a \neq 0$.
> - vertex; the maximum or minimum point of a parabola; the axis of symmetry of a parabola passes through this point; coordinates are $(-b/(2a),y)$
>
> In addition, there are also 8 other cards which can be matched in 4 pairs, such as in a traditional memory game.
>
> - quadratic function with one root and $a > 0$; (corresponding graph)
> - quadratic function with two roots and $a < 0$; (corresponding graph)
> - quadratic function with one zero and $a > 0$; (corresponding graph)
> - quadratic function with two zeros and $a < 0$; (corresponding graph)
>
> [2] **Memory Game with Connections:** Instead of just being a conventional memory game, this is now a *memory game with connections*. Players not only need to recall facts (such as definitions) from memory, but for some groups they also need to exert some cognitive effort in finding an appropriate match.
>
> An advantage of the memory game with connections is that it should get students to discuss whether or not two cards are a match for each other. If challenged by another student in the group, the student claiming the matched pair for himself (or herself) has to explain to the other students in his (or her) group why those two cards are a match. So it should create some discussion between the students.

roots	**zeros**
solutions	**x-intercepts**
parabola	**you-vee shape**
graph of $y = ax^2 + bx + c$, $a \neq 0$	**curve of a 2^{nd}-degree equation**
vertex	**the maximum or minimum point of a parabola**

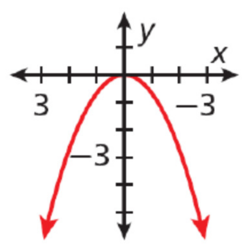
the axis of symmetry of a parabola passes through this point

coordinates are (-b/(2a), y)

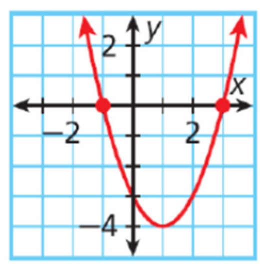
quadratic function with one root, and a > 0



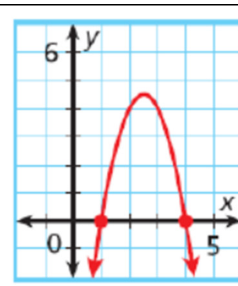
quadratic function with one zero and a < 0



quadratic function with two zeros and a > 0



quadratic function with two roots and a < 0



Name ______ Date ______ Class ______

LESSON 9-4

Practice A

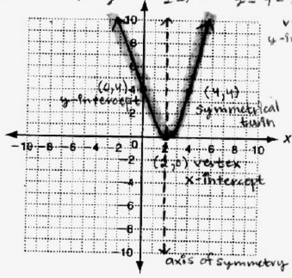
Solving Quadratic Equations by Graphing

Solve each quadratic equation by graphing the related function.

1. $x^2 - 4x + 4 = 0$ $x = \frac{-b}{2a}$ $a = 1, b = -4$

$x^2 - 4x + 4 = y$ $\frac{4}{2} \Rightarrow 2$ $y = 4 - 8 + 4$ $y = 0$

vertex $= (2, 0)$
y-intercept $= (0, 4)$



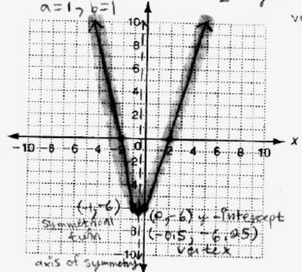
The solution is 2.

2. $x^2 + x = 6$ $x^2 + x - 6 = y$ $y = \frac{1}{4} - \frac{2}{4} - \frac{24}{4}$ $y = -\frac{25}{4}$

$x = \frac{-b}{2a}$ $x = -\frac{1}{2}$ $y = -6\frac{1}{4}$

$a = 1, b = 1$

vertex $= (-0.5, -6.2$



-3 and 2

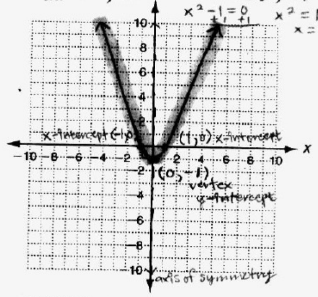
The solutions are -2 and 2.

Find the roots of each quadratic polynomial.

3. $x^2 - 1 = y$ $\frac{0}{2} \Rightarrow 0$ $y = 0 - 1$ $y = -1$

$x = \frac{-b}{2a}$ $a = 1, b = 0$ vertex $= (0, -1)$

$x^2 - 1 = 0$, $x^2 = 1$, $x = 1$ or -1

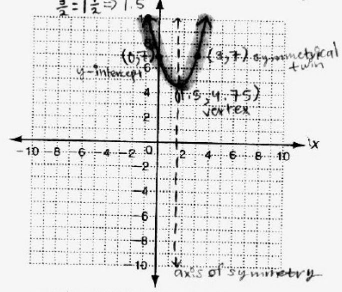


The roots are -1 and 1.

4. $x^2 - 3x + 7 = y$ $\frac{9}{4} - \frac{9}{2} + \frac{28}{4} \Rightarrow -\frac{9}{4} + \frac{28}{4} \Rightarrow \frac{19}{4} \Rightarrow 4\frac{3}{4} \Rightarrow 4.$

$x = \frac{-b}{2a}$ $a = 1, b = -3$ vertex $= (1.5, 4.75)$

$\frac{3}{2} = 1\frac{1}{2} \Rightarrow 1.5$



There are no roots.

5. Gretchen throws a ball straight up in the air. The quadratic function $y = -16x^2 + 48x$ models the height in feet of the ball after x seconds. Use a graphing calculator to sketch the graph of this function. Use the zeros to find how long the ball is in the air.

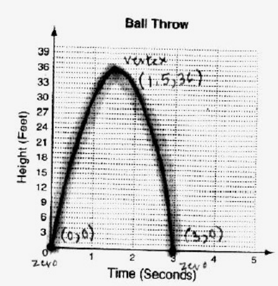
The ball is in the air for 3 seconds.

$y = -16x^2 + 48x$ $x = \frac{-b}{2a}$ $a = -16, b = 48$

$\frac{-48}{-32} \Rightarrow \frac{12}{8} \Rightarrow \frac{3}{2}$ $y = \frac{-16}{1}(\frac{9}{4}) + \frac{48}{1}(\frac{3}{2})$

$y = -36 + 72$

$y = 36$ vertex $= (1.5, 36)$



Name ____________ Date ________ Class ________

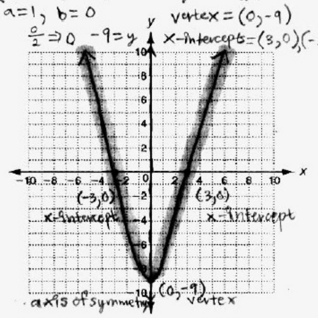
LESSON **9-4**

Practice C

Solving Quadratic Equations by Graphing

Solve each quadratic equation by graphing the related function.

1. $x^2 - 9 = 0$ $x^2 - 9 = y$ $x = \frac{-b}{2a}$
$a = 1,\ b = 0$ $\frac{0}{2} \Rightarrow 0$ $-9 = y$ vertex $= (0, -9)$ x-intercepts $= (3,0)(-3,0)$



The solutions are −3 and 3.

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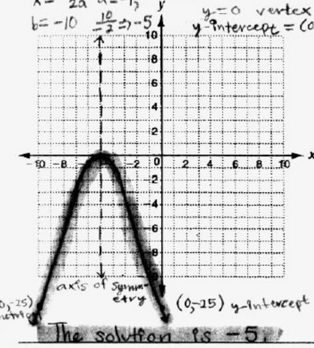
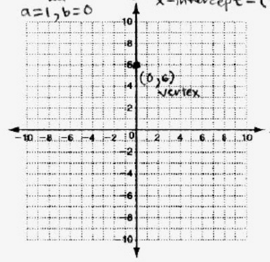
2. $-x^2 - 10x = 25$ $-x^2 - 10x - 25 = y$
$x = \frac{-b}{2a}$ $a = -1,$ $b = -10$ $\frac{10}{-2} \Rightarrow -5$ $-25 + 50 - 25 = y$ $y = 0$ vertex $= (-5, 0)$ y-intercept $= (0, -25)$



The solution is −5.

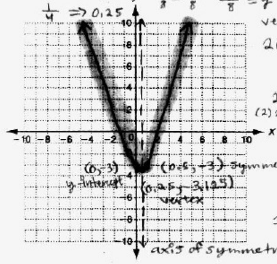
Find the roots of each quadratic polynomial.

3. $x^2 + 6 = y$ $\frac{0}{2} \Rightarrow 0$ $y = 6$ $x^2 = -6$
$x = \frac{-b}{2a}$ $a = 1, b = 0$ vertex $= (0, 6)$ x-intercept $= (?, 0)$



There are no real roots.

4. $2x^2 - x - 3 = y$ $\frac{1}{8} - \frac{1}{4} - 3 = y$ $-\frac{25}{8} = y$
$x = \frac{-b}{2a}$ $a = 2, b = -1$ $\frac{1}{8} - \frac{2}{8} - \frac{24}{8} = y$ $-3\frac{1}{8} \Rightarrow -3.125$
$\frac{1}{4} \Rightarrow 0.25$ vertex $= (0.25, -3.125)$
$2(2)^2 - 2 - 3 = 0$
$8 - 2 - 3 \neq 0$
$2(-1.5)^2 + 1.5 - 3 = 0$
$(2)2.25 - 1.5 - 3 \neq 0$
$2(-1)^2 + 1 - 3 = 0$
$2 + 1 - 3 = 0$ $(-1, 0)$
$2(1)^2 - 1 - 3 = 0$
$2 - 1 - 3 \neq 0$
$2(1.5)^2 - 1.5 - 3 = 0$
$4.5 - 1.5 - 3 = 0$
$3 - 3 = 0$ $(1.5, 0)$

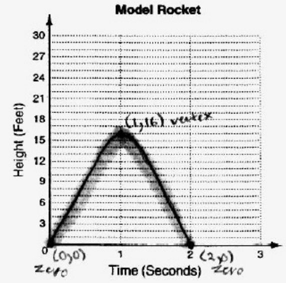


The roots are −1 and 1.5.

5. A model rocket is launched into the sky. The quadratic function $y = -16x^2 + 32x$ models the height in feet of the rocket after x seconds. How long is the rocket in the air?

The rocket is in the air for 2 seconds.

$y = -16x^2 + 32x$ $x = \frac{-b}{2a}$ $a = -16, b = 32$
$\frac{-32}{-32} \Rightarrow 1$ $y = 16$ vertex $= (1, 16)$



Egg Launch Contest (Part 1)

Vista View students are holding an egg launching contest on the field. Teams of students have built catapults that will hurl an egg down the field.

Team A: Mrs. Hoang's math students used their catapult and hurled an egg down the football field. They used a motion detector to collect data while the egg was in the air. They came up with the table of data below.

Distance (x) of egg from starting point (in feet)	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Height (y) of egg from the ground (in feet)	**0**	**7**	**12**	**15**	**16**	**15**	**12**	**7**	**0**

Team B: Mrs. Preciado's math students' egg flew through the air and landed down the field. The students tracking the path of the egg determined that the equation

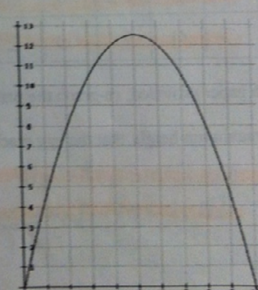
$$y = -2.0x^2 + 12x + 0$$

-32 + 48 -50 + 60 -12+72=0 -98 + 98 =

represents the path the egg took through the air, where x is the distance from the starting point and y is the height of the egg from the ground. (Both measurements are in feet.)

Team C: Mrs. Photoglou's math students launched an egg with their catapult. His students created a video of the trajectory of the egg and found that the graph to the right shows the path of the egg.

The x-axis represents the distance (in feet) of the egg from the starting point. The y-axis represents the height (in feet) of the egg from the ground.



Egg Launch Contest (Part 1)

Team A

1. On the graph paper provided, graph the path of Team A's egg.

2. What is the maximum height that the egg reached? How far was the egg hurled?

The maximum height that the egg reached was 16 feet and the egg went 8 feet.

Team B

3. Using the equation from Team B, generate a table of values that shows different locations of the egg as it flew through the air.

Distance (x)	0	1	2	3	4	5	6	7			
Height (y)	0	10	16	18	16	10	0	-4			

4. On the graph paper provided, graph the path of Team B's egg.

5. What is the maximum height that the egg reached? How far was the egg hurled?

the maximum height the egg went is 18ft and it hurled 6 feet.

Team C

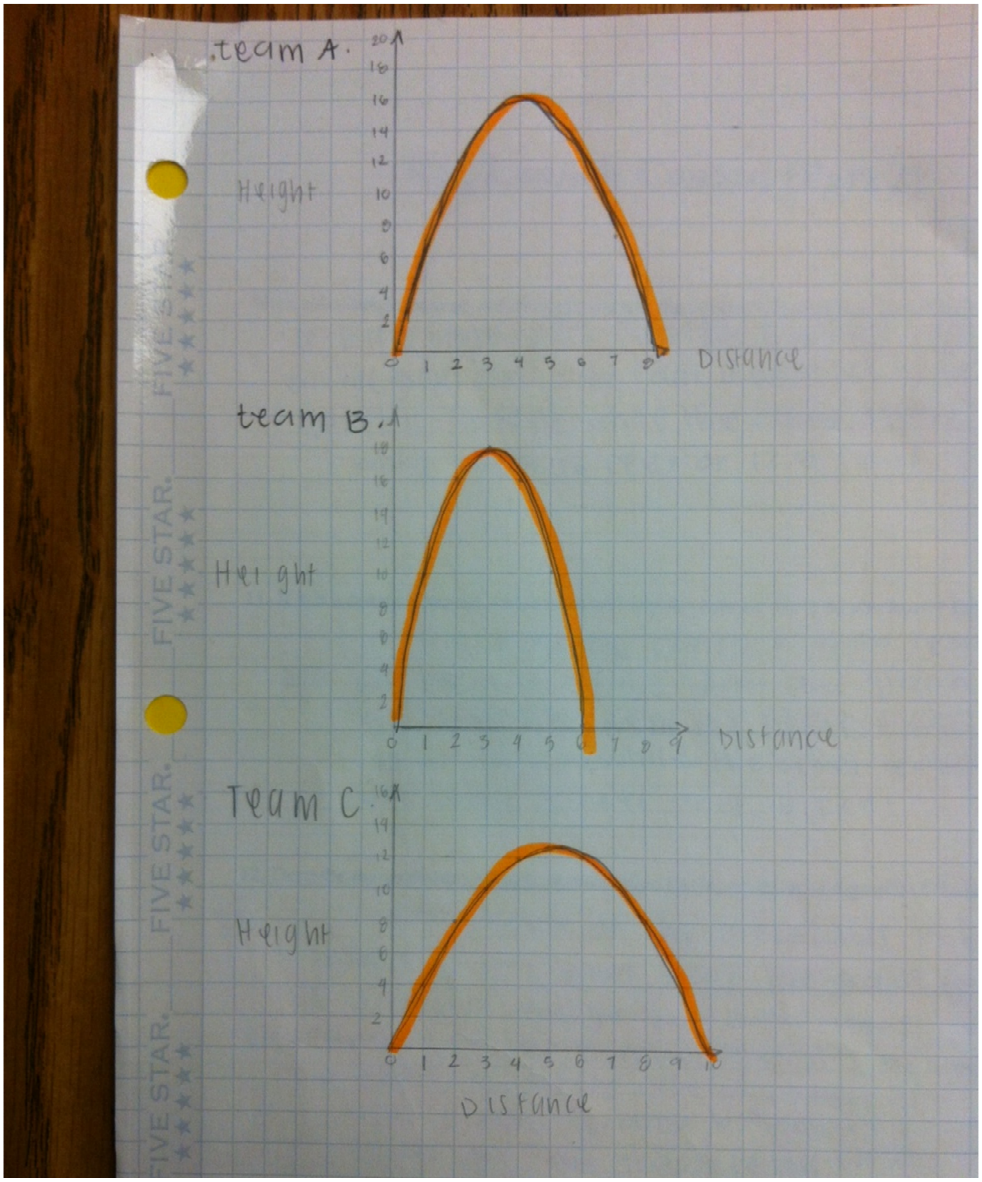
6. Using the data from Team C, generate a table of values that shows different locations of the egg as it flew through the air.

Distance (x)	0	1	2	3	4	5	6	7	8	9	10	
Height (y)	0	4	8	10	12	12.5	12	10.5	8	4	0	

8. On the graph paper provided, re-graph the path of Team C's egg.

9. What is the maximum height that the egg reached? How far was the egg hurled?

The maximum height the egg reached is 12.5 feet and the egg went 10 feet.


team A.
Height
Distance
team B.
Height
Distance
Team C.
Height
Distance

Egg Launch Contest (Part 2)

10. If it is a height contest, which team will win the contest? How do you know? Explain.

If it is a height contest, Then team B will win because on the graph and on the chart it went the highest which is 18 feet, while the other teams only went 16 feet or 12.5 feet.

11. If it is a distance contest, which team will win the contest? How do you know? Explain.

If it is a distance contest, then team C will win because on the chart and graph, their egg launcher went the furthest with is 10 feet, while the other teams only went 8 feet and 6 feet.

12. Describe the usefulness of <u>each</u> representation (table, equation, and graph) of the data.

I think each of the meathods is all useful in it's own way. The graph gives us a visual image but sometimes you can't see the numbers and it's not accurate. The chart is pretty useful, when you use x and y, and because of the chart you can graph realy easily. the equation is useful because you can sub in x-values and solve for y which is the one you want to find out, but if the equation is long and hard it's very time consuming.