LESSON OVERVIEW

Overview:

In this lesson students write the general expression for a pattern that is growing quadratically. They use that general expression to determine the number of tiles in any pattern in the sequence. Students then compare various equivalent quadratic expressions and simplify them to demonstrate that they are algebraically equivalent. This requires students to make connections between and among the expressions and the picture or table representation of the relationship of the place of the pattern in the sequence and the number of tiles in the pattern; and to evaluate and factor quadratic expressions.

CA Standards Addressed:

10.0 Students add, subtract, multiply and divide monomials and polynomials. Students solve multi-step problems, including word problems, using these techniques.

14.0 Students solve a quadratic equation by factoring or completing the square.

23.0 Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity.

24.0-25.0 The reasoning standards are in italics within the algebra standards. When mathematical reasoning is expected in the lesson the text will be labeled Mathematical Reasoning within the text.

Mathematical Goals of the Lesson:
Students will determine the expression for the number of tiles in any pattern in the sequence.
Students will interpret quadratic expressions in terms of the problem.
Students will solve the problem using a variety of strategies
Students will demonstrate the equivalence of various quadratic expressions by adding, subtracting, multiplying or dividing monomials and polynomials.
Students will justify their solutions to the problem.

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Access Strategies: Throughout the document you will see icons calling out use of the access strategies for English Learners, Standard English Learners, and Students With Disabilities.

Access Strategy	Icon	Description
Cooperative and Communal Learning Environments	CLE	*Supportive learning environments that motivate students to engage more with learning and that promote language acquisition through meaningful interactions and positive learning experiences to achieve an instructional goal. Working collaboratively in small groups, students learn faster and more efficiently, have greater retention of concepts, and feel positive about their learning.*
Instructional Conversations	IC	*Discussion-based lessons carried out with the assistance of more competent others who help students arrive at a deeper understanding of academic content. ICs provide opportunities for students to use language in interactions that promote analysis, reflection, and critical thinking. These classroom interactions create opportunities for students' conceptual and linguistic development by making connections between academic content, students' prior knowledge, and cultural experiences*
Academic Language Development	ALD	*The teaching of specialized language, vocabulary, grammar, structures, patterns, and features that occur with high frequency in academic texts and discourse. ALD builds on the conceptual knowledge and vocabulary students bring from their home and community environments. Academic language proficiency is a prerequisite skill that aids comprehension and prepares students to effectively communicate in different academic areas.*
Advanced Graphic Organizers	GO	*Visual tools and representations of information that show the structure of concepts and the relationships between ideas to support critical thinking processes. Their effective use promotes active learning that helps students construct knowledge, organize thinking, visualize abstract concepts, and gain a clearer understanding of instructional material.*

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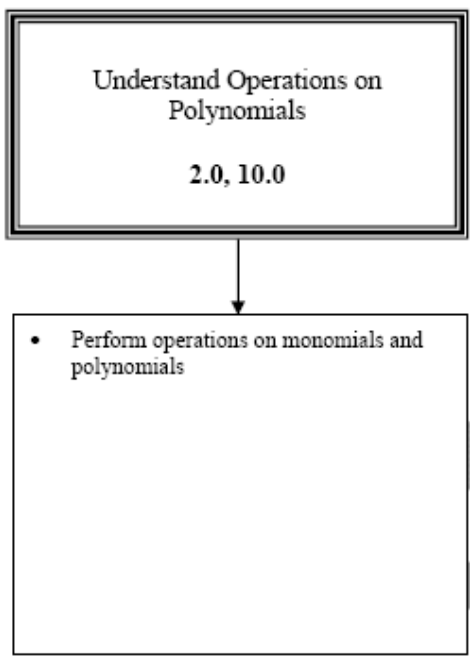
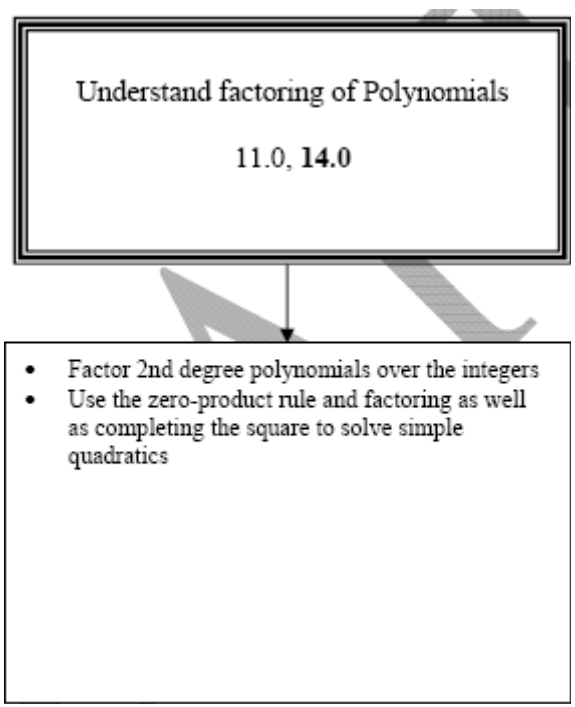
Academic Language Goals of the Lesson:

Assumption of Prior Knowledge:	**Academic Language:**	**Materials:**
Students express quantitative relationships by using algebraic terminology, expressions, and equations. Use variables and appropriate operations to write an expression and an equation. Use the correct order of operations to evaluate algebraic expressions such as 3(2x + 5)2. Simplify numerical expressions by applying properties of rational numbers (e.g., distributive, associative, commutative) Use algebraic terminology (e.g., variable, equation, term, coefficient, inequality, expression, constant) correctly.	Quadratics Quadratic equation Pattern Equivalent quadratic expressions Factor quadratic expressions	• S-pattern task (attached); square tiles • Four Fold Recording Sheet • Straight-edge • Pencil • Paper

Connections to the LAUSD Algebra 1, Unit _____, Instructional Guide





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Key:
Suggested teacher questions are shown in bold print.
Questions and strategies that support ELLs are underlined identified by an asterisk.
Possible student responses are shown in italics

CLE	Students work in groups as they Explore the problem
GO	Students use a table to present their work

Phase	SET UP PHASE: Setting Up the Mathematical Task—Part 1	
S E T U P	**INTRODUCING THE TASK** Prior to the lesson: • arrange the desks so that students are in groups of 4. • determine student groups prior to the lesson so that students who complement each other's skills and knowledge core are working together. • place materials for the task at each grouping. • solve the task yourself. HOW DO I SET-UP THE LESSON? Ask students to follow along as you read the problem. Then have several students explain to the class what they are trying to find when solving the problem. Stress to students that they will be expected to explain how and why they solved the problem a particular way and to refer to the context of the problem. Clarify any confusions students may have but do not suggest specific values for their investigation. • Give the students individual time to think about this question and begin a class discussion • *A Frayer model could be used to assist students in developing a clear understanding of some of the terms to be used in the lesson. • *If the students are NOT able to easily answer the first question then it may be appropriate (especially for EL, SEL and SWD) to spend time doing an additional hands on activity (simple linear pattern) to reinforce concept.	Students will be more successful in this task if they understand what is expected in terms of group work and the final product. It is critical that you solve the problem in as many ways as possible so that you become familiar with strategies students may use. This will allow you to better understand students' thinking. As you read through this lesson plan, different strategies for solving the problem will be given. HOW DO I SET-UP THE LESSON? As students describe the task, listen for their understanding of the goals of the task. It is important that they indicate the goal is to determine the expression for the number of tiles in any pattern in the sequence. Be careful not to tell students how to solve the task or to set up a procedure for solving the task because your goal is for students to do the problem solving.

To assist ELLs' participation in the class discussion*:
- Allow time for students to first talk in small groups (pairs) and then have the groups report to the whole class.*
- Reinforce appropriate language as students communicate their ideas (e.g. re-voice a student's contribution in complete, grammatically correct language). Ask students if you have captured what they said*.
- Create work groups that are heterogeneous according to language proficiency*
- Model appropriate mathematical language, emphasizing vocabulary used in appropriate context.*

Phase	EXPLORE PHASE: Supporting Students' Exploration of the Task STRUCTURE	
E X P L O R E E X P L O R E E X P L O R E	**PRIVATE THINK TIME** Give students 5 - 7 minutes of private think time to begin to solve the problem individually. Circulate among the groups assessing students' understanding of the idea below. **FACILITATING SMALL GROUP RPOBLEM SOLVING** • After about 15 minutes, ask students to work with their partner or in their small groups to discuss what they discovered. • As you circulate among the groups, press students to come up with more than one solution and then show that the expressions are equivalent. After explaining their initial solutions you might say "**Now look at the figures again." Find another way to look at how the pattern is growing from one figure to the next.** As you circulate among the groups, press students to come up with more than one solution and then show that the expressions are equivalent. After explaining their initial solutions you might say **Now look at the figures again. Find another way to look at how the pattern is growing from one figure to the next.** After there is more than one solution in the group press student to show that the expressions are equivalent algebraically by using their prior knowledge of various properties such as the distributive property, properties of exponents, and rules for adding and subtracting like terms. *What do I do if students have difficulty getting started?* Allow students to work in their groups to solve the problem. Assist students/groups who are struggling to get started by prompting with questions such as: ▪ **What do you notice about the figures in the pattern?** ▪ **How many tiles are in the first figure? The second figure? the fifth figure?**	**PRIVATE THINK TIME** Make sure that students' thinking is not interrupted by talking of other students. If students begin talking, tell them that they will have time to share their thoughts in a few minutes. **FACILITATING SMALL GROUP RPOBLEM SOLVING** The teacher's role when students are working in small groups is to circulate and listen with the goal of understanding students' ideas and asking questions that will advance student work. • Be persistent in asking questions related to the mathematical ideas (see question suggestions in the following section), exploration strategies, connections between representations. • Be persistent in asking students to explain their thinking and reasoning. • Be persistent in asking students to explain, in their own words, what othe students have said. • Be persistent in asking students to use appropriate mathematical languag *What do I do if students have difficulty getting started?* By asking a question such as "What do you notice about the figures pattern?" the teacher is providing students with a question that can be used over and over when problem solving. This will help them focus on what they know, what they were given, and what they need to determine. *What misconceptions might students have?* Misconceptions are common. Students may have learned the information incorrectly or they may generalize ideas prematurely. Some strategies for helping students discover when they have made an error include: - Ask students to extend a pattern and compare it to what they predicted pattern sh

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- **How are the figures changing from one to the next?**
- Give students square tiles and ask them to construct the 1st, 2nd, and 3rd figures. Then ask: **How did you know how to construct the figures?**

How would you construct the 4th figure?

What misconceptions might students have?
Look for and clarify any misconceptions students may have.

a. *looking only at the first two figures and assuming it's a linear growth pattern.* **How many tiles are in the third figure? The fourth figure? Does this match your pattern?**

Which problem-solving strategies might be used by students? How do I advance students' understanding of mathematical concepts or strategies when they are working with each strategy?

Students will approach the problem using a variety of strategies. Some strategies are shown below. Questions for assessing understanding and advancing student learning are listed for each.

Using the drawing

A. *moving a row and making it a column to form a square plus one tile*

How do the dimensions of the square compare with the figure number? What would the dimensions for the 100th square be? How would you draw the nth figure? How would you write an expression for the nth figure?

Which problem-solving strategies might be used by students? How do I advance students' understanding of mathematical concepts or strategies when they are working with each strategy?

Using the drawing

A. T = (N • N) + 1 OR N_2 + 1

Figure 5 Move the bottom row and make it a column resulting in a 5 by 5 square plus one tile

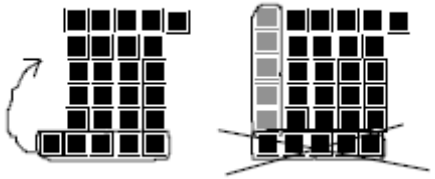
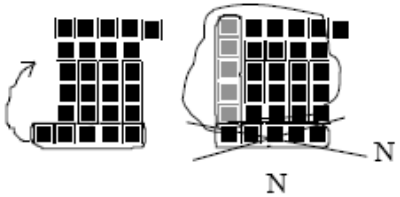
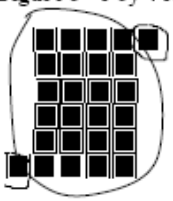


Figure N An N by N square plus one tile



B. T= (N + 1)(N - 1) + 2

Figure 5 6 by 4 rectangle + 2 tiles



B.. *-seeing each figure as a rectangle with 2 additional tiles at opposite corners*

How do the dimensions of the rectangle compare with the figure number? What would the dimensions for the 100th rectangle be? How would you draw the nth figure? How would you write an expression for the dimensions of the nth rectangle? The nth figure?

Simplify your expression.

Press students to use the properties and rules they have learned dealing with monomials and polynomials.

Ask them to connect the expression $N^2 + 1$ to the diagram.

C. - *seeing each figure as a square with a row of tiles 1 larger than the side of a square above the square and another row below the square*

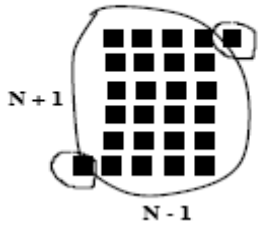
How do the dimensions of the square compare with the figure number? What would the dimensions for the 100th square be? What would the rows for the 100th figure look like? How would you draw the nth figure? How would you write an expression for the nth square? The nth figure?

Simplify your expression.

Students may use the "FOIL" method to arrive at $N^2 - N - N + 1 + 2N$ and then rules for combining like terms to simplify the expression to $N^2 + 1$. You might ask **Why does the FOIL method work? How does it connect to the distributive property? How did you know which terms to combine with each other?**

Ask them to connect the expression $N^2 + 1$ to the diagram.

Figure N (N+1) by (N-1) rectangle plus 2 tiles



C.. T = (N-1)(N-1) + 2N

Figure 5 4 by 4 square with 1 by 5 rows above and below

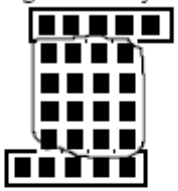
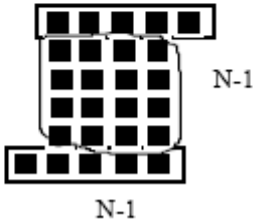


Figure N (N-1) by (N-1) square with 2 rows 1 by N



D. T = N(N+1) – (N-1).

Figure 5 a 5 by 6 rectangle with 4 tiles removed

X
x
X
x

Figure N an N by (N+1) rectangle with N-1 tiles removed

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D. *–moving the corner square tile to the beginning of its row and then seeing the figure as a rectangle from which tiles must be subtracted*

How do the dimensions of the rectangle compare with the figure number? What would the dimensions for the 100th rectangle be? How would you draw the nth figure? How would you write an expression for the dimensions of the nth rectangle? The nth figure?

Simplify your expression.

Press students to use the properties and rules they have learned dealing with monomials and polynomials.

Students should use the distributive property to arrive at $N_2 + N – (N – 1)$ and then the distributive property again to arrive at $N_2 + N – N + 1$.They should then use rules for combining like terms to simplify the expression to $N_2 + 1$. You might ask **How does the distributive property work?**

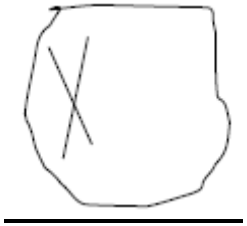
Ask them to connect the expression $N_2 + 1$ to the diagram.

Using a table

- *student constructs a table by counting the tiles*

What pattern do you notice in the number of tiles? If students have previous experience with linear growth patterns, they may attempt to find a linear relationship. You could ask them **How is the number of tiles changing in the table?**

Press students to make the connection to a quadratic pattern. Then press them to connect the expression to the diagram. You might ask **So how can you see this pattern in the figures**?



Using a table

Figure no.	No. tiles	
1	2	3
2	5	5
3	10	7
4	17	9
5	26	

Figure no.	No. tiles	Pattern
1	2	1+1
2	5	4+1
3	10	9+1
4	17	16+1
5	26	25+1

MONITORING STUDENTS' RESPONSES

As you circulate, attend to students' mathematical thinking and to their conjectures, in order to identify those responses that will be shared during the Share, Discuss, and Analyze Phases.

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Sharing, Discussing, and Analyzing

Orchestrating the mathematical discussion: a possible Sequence for sharing student work, Key Questions to achieve the goals of the lesson, and possible Student Responses that demonstrate understanding.

Revisiting the Mathematical Goals of the Lesson:

The purpose of this sharing/discussion is to make explicit the conclusions the exploration.

IC	Students discuss what they learned from the exploration and share their thinking

Phase	Sequencing of Student Work and Possible Questions	Rationale and Mathematical Ideas	Student Responses and Comments
S H A R E D I S C U S S A N A L Y Z E	**FACILITATING THE GROUP DISCUSSION** *What order will I have students post solution paths so I will be able to help students make connections between the solution paths?* As you circulate among the groups, look for solutions that will be shared with the whole group and consider the order in which they will be shared. Ask students to explain their solutions to you as you walk around. Make certain they can make sense of their solutions in terms of the diagram. The work of at least two groups should be chosen. Look for a variety of strategies used by the students to develop and write their rules, such as "*Newton's difference*" approach, and the once shown above in the Explore phase. If the work can be displayed, ask students to post them in the front of the classroom, then have the class look at the students' work, without any explanation, and ask students other than those in the contributing groups how they think the problem has been solved. The goal is to discuss mathematical ideas associated with the quadratic expressions and their equivalence. If it is not possible to display the work to the whole class, have groups present and explain their own	All students should develop a rich understanding of the mathematical ideas associated with the quadratic expressions and their equivalence. Students should be able to make connections between different solution paths.	**FACILITATING THE GROUP DISCUSSION** *What order will I have students post solution paths so I will be able to help students make connections between the solution paths?* Even though you may display all solution paths, you should strategically pick specific solution paths to discuss with the whole group. For this particular problem it could be best to have the solution $N^2 + 1$ shared first so that all other expressions can be simplified to that. If no one has come up with that solution you could say **This is the solution I came up with.** Then demonstrate the solution's connection to the diagram. **Is my expression equivalent to yours? Show me.** Recognizing equivalent forms of expressions and being able to convert flexibly among them means that a student should be able to write a polynomial in factored form. That is, a student should understand that $x2 + 7x + 10 = (x + 2)(x + 5)$. Further, students should recognize that both expressions represent a quadratic function that crosses the x-intercept at (2, 0) and (-5, 0). This should become evident once they begin to graph them.

S H A R E D I S C U S S A N A L Y Z E	*What question can I ask throughout the discussion that will help students keep the context and the goal of the problem in mind? (Driving Questions *)* *N_2 + 1 solution* If students have come up with the solution, N_2 + 1, ask them to present their solution and explain how their solution connects to the diagram. You might ask **What does the N_2 look like in the diagram? Where do you see the "1"?** Ask students who have come up with a different solution to present their solutions: *T= (N + 1)(N - 1) + 2 solution* Ask students to present their solution and explain how it connects to the diagram. Driving Question *** How is your solution equivalent to N_2 + 1? Show us.** Make certain to press on the same questions noted above for this solution. *T = (N-1)(N-1) + 2N solution* Ask students to present their solution and explain how it connects to the diagram. Driving Question *** How is your solution equivalent to N_2 + 1? Show us.** Make certain to press on the same questions noted above for this solution. *T = N(N+1) – (N-1) solution* Ask students to present their solution and explain how it connects to the diagram. Driving Question *** How is your solution equivalent to N_2 + 1? Show us.** Make certain to press on the same questions noted above for this solution. *Table Solution* Ask students to demonstrate how they arrived at the		*What question can I ask throughout the discussion that will help students keep the context and the goal of the problem in mind? Driving questions (*)* Driving Questions have been provided because they will help to stimulate student interest, maintain the focus of the discussion on the problem context, and focus the discussion on key mathematical ideas. Many of the questions require students to take a position or to wonder about mathematical ideas or problem solving strategies. Accountable Talk *Pointing to the Diagram* Asking student to point to the diagram and mark their method of seeing the relationship as they explain their solutions *Repeating or Paraphrasing Ideas* Ask other students to put explanations given by their peers into their own words. This is a means of assessing understanding and providing others in the class with a second opportunity to hear the explanation. *Position-Driven Discussion* Press students to take a position and to support their claims with evidence from the diagrams. Students must say why they believe their expression describes the pattern. In doing so they will have to provide reasons for their claims. Mathematical Reasoning: *Connecting to* the distributive property they should be able to indicate that the expression in the first set of parenthesis must be distributed across the expression in the second set of parenthesis. *The distributive property* - Students should explain that the term "outside"of the parenthesis must be distributed or multiplied times each term "inside"

S H A R E D I S C U S S A N A L Y Z E	table and explain how it connects to the diagram. **<u>HOMEWORK</u>** You could give the Extend Pattern of Tiles problem as a homework assignment.		the parenthesis. *Combining like terms* – Students should explain that only terms that are the same can be added or subtracted . For example, N2 does not mean the same thing as N and therefore the two cannot be combined. *<u>HOMEWORK</u>* *In addition, students could work on problems like the one below to make* *connections to real life applications of factoring polynomials.* Measure the length and width of a rectangular room in your home, to the nearest foot. Suppose you want to buy a carpet to fit in the room with a space *x* feet wide on all four sides. Find a model for the area of the rug. Write it as a quadratic trinomial. Use the model to find the cost of the carpet if x = 3 and carpet costs $7.50 a square foot.

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